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# INERTIA-LIMITED COMPACTION OF A POROUS MEDIUM BY A GAS PISTON

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The action of a high-pressure gas on an adjacent porous medium is considered. The gas penetrates into the pore space and shifts the medium's particles, leading to compression of the porous bed. In this paper, we study the limiting case of a medium with a low mechanical strength which is almost completely compacted at a certain moment. The dynamics of filtration and compression of the porous structure is studied analytically (at the qualitative level) and numerically.

Initiation of a porous explosive by gas detonation or electric discharge [1] exemplifies phenomena for which the interaction of filtration flow and compaction is significant. The same processes are typical of the initial stages of explosion in porous ground [2–4]. A similar situation is also possible in explosive compaction of powders.

**Formulation of the Problem.** At initial time ($t = 0$), the half-space $x < 0$ is occupied by a gas with initial pressure $p_0$ and density $\rho_0$. A medium with initial open porosity $\varphi_0$ is located in the half-space $x > 0$. The gas pressure in the pores is small compared with $p_0$. The initial medium's particle diameter is $d$, and the particle density $\rho_s$ is considerably higher than $\rho_0$. The particle's strength is ignored, i.e., the powder is compacted without resistance.

Under these assumptions, the interaction of the gas and the porous medium is divided into two stages. Initially the gas is filtered into the porous medium, which is practically at rest at this stage because of the significant difference between the densities. The solution of the problem of sudden filtration into the static medium can be found using some additional simplifications.

Then, the particles' displacement by the gas flow becomes significant. In the absence of mechanical strength, this leads to the formation of a compact "plug" near the surface of the porous medium. At least at the beginning of particle acceleration, particle motion can be found under the assumption that the state of the gas phase is known from the solution of the filtration problem. Of course, for appreciable deformation of the medium, this approximation is not valid, but it can be used to estimate integral parameters such as the time of compaction.

In this paper, we discuss the interaction of phases for a two-velocity model of the medium. Next, we consider analytically the two initial stages of the processes and compare the estimates with a numerical solution in a two-phase formulation. Fair agreement is obtained.

**Equations and Closing Relations.** The standard system of equations for two-phase flow has the form [5, 6]

$$\frac{\partial \rho\varphi}{\partial t} + \frac{\partial \rho\varphi u}{\partial x} = 0, \quad \frac{\partial \rho\varphi u}{\partial t} + \frac{\partial \rho\varphi u^2}{\partial x} + \varphi\frac{\partial p}{\partial x} = -f,$$
$$\frac{\partial \rho\varphi E_g}{\partial t} + \frac{\partial \rho\varphi u E_g}{\partial x} + p\frac{\partial(\varphi u + \alpha v)}{\partial x} = f(u - v) - q,$$
$$\rho_s\Big(\frac{\partial \alpha}{\partial t} + \frac{\partial \alpha v}{\partial x}\Big) = 0, \quad \rho_s\left(\frac{\partial \alpha v}{\partial t} + \frac{\partial \alpha v^2}{\partial x}\right) + \alpha\frac{\partial p}{\partial x} + \frac{\partial p_s}{\partial x} = f, \qquad (1)$$

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$$\frac{\partial E_s}{\partial t} + v\frac{\partial E_s}{\partial x} + \frac{p_s}{\rho_s \alpha}\frac{\partial v}{\partial x} = q,$$

$$p = (\gamma - 1)\rho E_g = \rho RT/\mu, \quad E_s = c_p(T_s - T_0) + E_p.$$

Here $\rho$ is the gas density, $\rho_s$ is the particle density (constant), $\alpha$ is the volume fraction of the solid phase, $\varphi$ is the porosity ($\alpha + \varphi = 1$), $u$ is the gas-flow velocity, $v$ is the solid-phase velocity, $E_g$ and $E_s$ are the internal energies of the gas and the solid phase, $p$ is the pressure, $T$ and $T_s$ are the gas and solid-phase temperatures, $c_p$ is the particle heat capacity, $f$ is the force of interphase friction, $q$ is the heat exchange, and $p_s$ is the solid pressure (it was not considered in the analytical solution, but was taken into account in some numerical calculations). The energy $E_s$, generally speaking, can contain an elastic term $E_p$. One possible variant of the equation of state of the solid phase is given in [1]. In this work, we do not specify the particular types of relations for $p_s$ and $E_p$, since their roles are insignificant for a medium without strength.

The friction force $f$ is given by the standard Ergun relation

$$f = C_f\frac{\alpha}{\varphi}\frac{\rho(u - v)|u - v|}{d}.$$

This correlation holds for the fairly high relative velocities considered in the present paper. The resistance coefficient $C_f$ is smaller than the value of 1.75 in [7] by a factor of two, in agreement with the latest data [8].

In the numerical calculations, we used Denton's formula [9] for the heat exchange $q$ between the phases:

$$q = \lambda \mathrm{Nu}\frac{6\alpha}{d}\frac{T - T_s}{d}, \quad \text{where} \quad \mathrm{Nu} = \left(\frac{\rho d|u - v|\varphi}{\eta}\right)^{0.7}.$$

Compaction of a brittle powder is accompanied by particle fragmentation. As the porosity $\varphi$ decreases, the initial particle size $d$ (which enters into the formula for $f$ and $q$) is replaced by a size spectrum, which varies upon compression.

In analytical estimation, ignoring the effect of fragmentation is allowed. This is obvious for the problem of filtration into a static medium. The stage of particle acceleration is considered, in principle, at the qualitative level, and allowance for particle-size variation will not improve markedly the approximation.

In numerical solutions of complete system (1), it is certainly desirable to take fragmentation into account. At present, unfortunately, there are no generally accepted laws of friction and heat exchange that would describe this effect. The dependence on the porosity in Ergun's and Denton's formulas reflects the influence of different packing density of the monodisperse particles and is verified for a narrow range which does not include the compaction region of the powder.

Within the framework of a two-phase model, one can take into account fragmentation by introducing a variable characteristic particle size $d_{eff}$ [1, 10]. According to the estimate of [1], the relation $d_{eff} \sim \varphi$ is a reasonable approximation. Therefore, it is believed that Ergun's formula with fixed $d$ containing the product $\varphi d$ in the denominator reflects qualitatively the particle-size reduction due to compaction.

Thus, the applicability of full system (1) is limited because of the absence of detailed information on the interaction of the phases. However, calculation using this system allows one to estimate the role of additional simplifications used in the analytical approach.

**Filtration Stage.** At this stage, we ignore solid-phase motion, and general system (1) is reduced to the equations for the gas. Since the friction in the porous medium is a dominant factor, it is possible to omit translation terms in the equation of gas momentum and assume that the pressure gradient is balanced by the friction force: $\varphi \partial p/\partial x = -f$.

Indeed, the terms $\partial \rho\varphi u/\partial t$ and $\partial \rho\varphi u^2/\partial x$ have order $\rho\varphi u^2/L$, where $L$ is the characteristic flow scale. The friction force is $f \sim \rho u^2/d$. An averaged description of flow is possible when $L \gg d$. Consequently, both inertial terms are small compared with $f$.

In addition, because of the approximate estimate-oriented approach, it is reasonable to use a subsequent simplification of the problem. In place of the equation for the gas energy, we shall adopt the simplest adiabatic law $p \sim \rho^\gamma$.

For the filtration step, it is convenient to transform to the Lagrangian coordinate $r$, which is related to the Eulerian coordinate $x$ by the usual relation $\rho_0 dr = \rho dx$. Then, in place of (1), we obtain

$$\frac{\partial(1/\rho)}{\partial t} = \frac{1}{\rho_0}\frac{\partial u}{\partial r}, \quad \frac{\partial p}{\partial r} = -\frac{\beta \rho_0 u^2}{d}, \quad p = p_0\left(\frac{\rho}{\rho_0}\right)^\gamma. \tag{2}$$

Here $\beta = C_f\alpha/\varphi^2$ is a constant coefficient in the adopted approximation. The quantities $\rho$ and $u$ can be eliminated from (2). Denoting the initial isothermal speed of sound in the gas by $c = \sqrt{p_0/\rho_0}$, we have one equation for the dimensionless pressure $P = p/p_0$:

$$-\frac{1}{\gamma P^{1+1/\gamma}}\frac{\partial P}{\partial t} = c\sqrt{\frac{d}{\beta}}\frac{\partial}{\partial r}\sqrt{-\frac{\partial P}{\partial r}}. \tag{3}$$

This equation holds inside the porous medium over the time-dependent interval of the Lagrangian coordinate $R(t) < r < 0$. The left boundary of the interval $r = R(t)$ corresponds to the Eulerian coordinate $x = 0$, i.e., to the gas just entering the pores at this time. The right boundary $r = 0$ is the leading edge of the filtration wave, and its Eulerian coordinate will be defined below. The boundary condition on the right is $P = 0$ for $r = 0$, i.e., the initial gas content of the pores is ignored. On the left boundary of the gas penetrating into the pores, the pressure is considered constant: $P = 1$ for $r = R(t)$. This is justified by the insignificant gas leak into the crowded space due to the large flow resistance.

Since the problem does not contain characteristic times and length (the particle size $d$ is a microscopic parameter, which is not a characteristic distance for the flow), it is natural to seek a self-similar solution of the form $P(r/t^n)$. The self-similarity index is $n = 2/3$, as is evident from the coefficient in Eq. (3). It is convenient to use the dimensionless self-similar coordinate $\xi = kr/t^{2/3}$, where $k = (16\beta/9c^2d)^{1/3}$. For $P(\xi)$ we finally have the equation

$$P'' = \frac{(-P')^{3/2}\xi}{\gamma P^{1+1/\gamma}}. \tag{4}$$

At first glance, transformation to the Lagrangian coordinates, which gives a problem with the unknown boundary for Eq. (3) is not advantageous. But for the self-similar coordinate $\xi$, the interval on which Eq. (4) is defined is fixed. To find this interval, we write the mass of the gas injected into the pores before time $t$ as

$$-\rho_0 R(t) = \int_0^t \rho_0 u(R(t),t)\,dt.$$

Expressing $u$ in terms of $P$, we have

$$u = c\sqrt{\frac{d}{\beta}}\sqrt{-\frac{\partial P}{\partial r}} = c\sqrt{\frac{d}{\beta}}\sqrt{-\frac{\partial P}{\partial \xi}}\sqrt{\frac{k}{t^{2/3}}}.$$

On the left boundary $\xi_0 < 0$, the derivative $\partial P/\partial \xi$ is fixed and we easily obtain

$$R(t) = -\frac{3c}{2}\sqrt{\frac{kd}{\beta}}\sqrt{-\frac{\partial P}{\partial \xi}}\,t^{2/3}$$

or

$$\frac{kR}{t^{2/3}} = \xi_0 = -2\sqrt{-\frac{\partial P}{\partial \xi}}\bigg|_{\xi_0}. \tag{5}$$

Equation (4) was solved numerically. We set the boundary value $\xi_0$ for which $P = 1$. Then, the start derivative $\partial P/\partial \xi$ was calculated from (5) and integrated until the integral curve reached one of the coordinate axes. The method of sequential approximation was used to choose $\xi_0$ such that the integral curve passes through the coordinate origin, i.e., the "right" boundary condition $P(0) = 0$ is satisfied.

Figure 1 gives integral curves for various values of the adiabatic exponent $\gamma$. The value of $\xi_0$ was only slightly dependent on $\gamma$. Thus, we have $\xi_0 = -1.4364$ for $\gamma = 3$, and $\xi_0 = -1.2367$ for $\gamma = 1$.

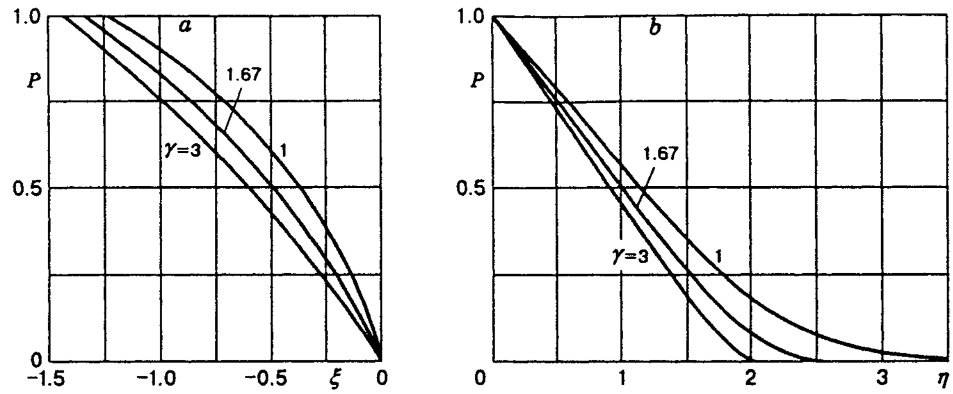


Fig. 1

For $\gamma > 1$, the solution at zero is described by a two-term power asymptotic relation: for $\xi \to 0$ we have

$$P = -A\xi - \frac{\gamma A^{1/2-1/\gamma}}{(2\gamma-1)(\gamma-1)}(-\xi)^{2-1/\gamma},$$

where the coefficient $A > 0$. For example, $A = 1.025$ for $\gamma = 3$ and $A = 1.67$ for $\gamma = 1.67$.

For $\gamma = 1$, the asymptotic relation is more complex. Direct calculations show that near zero we have $P = -(3/2)^{2/3}\xi \ln^{2/3}(-1/\xi)$. In this case, the derivative $\partial P/\partial\xi$ has a logarithmic singularity.

Calculation of particle motion requires transformation to the Eulerian coordinates. From the expression for the coordinate $x = r + \int\limits_0^t u(r,t)dt$, one readily obtains the following formula for conversion to the self-similar Eulerian coordinate $\eta = kx/t^{2/3}$:

$$\eta = -2\xi \int\limits_{\xi_0}^{\xi} \sqrt{-\frac{dP}{d\xi}}\frac{d\xi}{\xi^2} \equiv \int\limits_{\xi_0}^{\xi} P^{-1/\gamma} d\xi.$$

From the above asymptotic relation, it follows that the maximum value $\eta_f \equiv \eta(\xi = 0)$, i.e., the self-similar edge coordinate $\eta_f = 2\sqrt{A}$. Consequently, the filtration-wave edge moves as $x = 2\sqrt{A}t^{2/3}/k$. For $\gamma = 1$, a divergence of the form $\eta \sim \ln^{1/3}(-1/\xi)$ holds. In practice, this special case does not involve difficulties in numerical calculations, because the divergence is very weak. The graphs of $P(\eta)$ are shown in Fig. 1b.

**Compaction of the Porous Bed.** We now use the known filtration flow to estimate particle acceleration. The initial equation of motion

$$\rho_s\alpha\frac{dv}{dt} + \alpha\frac{\partial p}{\partial x} + \frac{\partial p_s}{\partial x} = f$$

is appreciably simplified if one ignores stresses in the solid phase and takes into account the equality $\varphi\partial p/\partial x = -f$:

$$\rho_s\alpha\frac{dv}{dt} = -\frac{\partial p}{\partial x}.$$

Next, because of the low particle velocity, one may not distinguish between the full and partial time derivatives. Then, the particle velocity is

$$v = \frac{3}{2}\frac{p_0 k}{\rho_s\alpha}t^{1/3}V(\eta), \quad \text{where} \quad V(\eta) = \sqrt{\eta}\int\limits_{\eta}^{\eta_f}\left(-\frac{\partial P}{\partial\eta}\right)\frac{d\eta}{\eta^{3/2}}. \tag{6}$$

Naturally, the velocity is maximal for particles at the interface $\eta = 0$. Compression of the medium is found

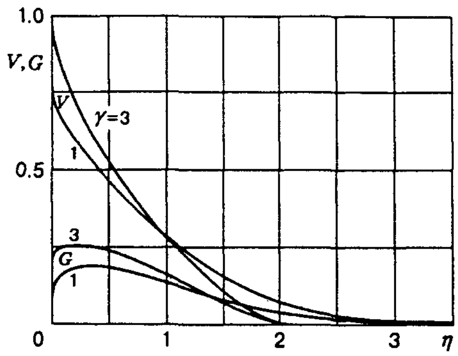


Fig. 2

from the velocity distribution:

$$\frac{\Delta V_s}{V_s} = \int_0^t \frac{\partial v}{\partial x}\,dt,$$

where $V_s$ is the specific powder volume. This equality is used to estimate the typical time of compression (the moment at which the pores are entirely closed, i.e., $\Delta V_s/V_s = \varphi_0$). Then, the compaction condition has the form

$$\varphi_0 = \frac{9}{4}\frac{p_0 k^2}{\rho_s \alpha_0} t^{2/3} F(\eta), \quad \text{where} \quad F(\eta) = \eta \int_\eta^{\eta_f} \Big(-\frac{\partial V}{\partial \eta}\Big)\frac{d\eta}{\eta^2}. \tag{7}$$

At first glance, in condition (7) one should set $\eta = 0$, i.e., consider the most intense compaction at the initial interface. Then, the time of collapse of the porous structure $t_c$ should be proportional to $p_0^{-3/2}$.

Numerical calculations using the full system of Eqs. (1), which are described in detail below, gave a different result. In reality, $t_c \sim p_0^{-1}$. It turns out that condition (7) agrees with the same dependence upon closer inspection.

The function $F(\eta)$ has a singularity of the form $\eta^{-1/2}$ near zero, which is derived from an analogous singularity of the derivative of the dimensionless velocity $dV(\eta)/d\eta$, as is readily seen from (6). Figure 2 shows the graphs of $V(\eta)$ and $G(\eta) = \sqrt{\eta}F(\eta)$ for extreme values of $\gamma = 1$ and 3. Evidently, in this interval of $\gamma$ we have $F \approx 0.15/\sqrt{\eta}$ for $\eta \to 0$.

Consequently, one cannot set $\eta = 0$ in (7). The divergence should be cut at a physically reasonable level. The natural range of applicability of continual equations to a porous medium is the spatial scale $d$ which is the initial particle size. Therefore, one should set $\eta = \eta_0 = kd/t^{2/3}$ and $F(\eta_0) \approx 0.15/\sqrt{\eta_0}$ in (7). Thus, we finally obtain

$$\varphi_0 \approx \frac{9\cdot 0.15}{4}\frac{p_0 k^{3/2}}{\rho_s \alpha}\frac{t_c}{\sqrt{d}} \quad \text{or} \quad t_c \approx 2\,\frac{d}{c}\frac{\varphi_0 \alpha_0 \rho_s}{\sqrt{\beta}\rho_0}. \tag{8}$$

Evidently, the time $t_c$ is actually inversely proportional to the initial pressure (i.e., density $\rho_0$).

The amount of the gas penetrating into the pores before the moment of collapse is also of interest. The mass $m_c = \rho_0 R(t_c) = \rho_0 \xi_0 t_c^{2/3}/k$ falls on a unit area of the interface. In the adopted approximations, we have

$$m_c \approx 2\rho_0 d\Big(\frac{\alpha_0 \varphi_0 \rho_s}{\beta \rho_0}\Big)^{2/3}. \tag{9}$$

**Numerical Simulation.** The analytical approach is qualitative. Numerical calculations give a more comprehensive estimate of the process.

System (1) subject to the above-described initial conditions was solved in Eulerian coordinates by the Lax–Wendroff method, which was modified to take into account the nondivergent and algebraic terms

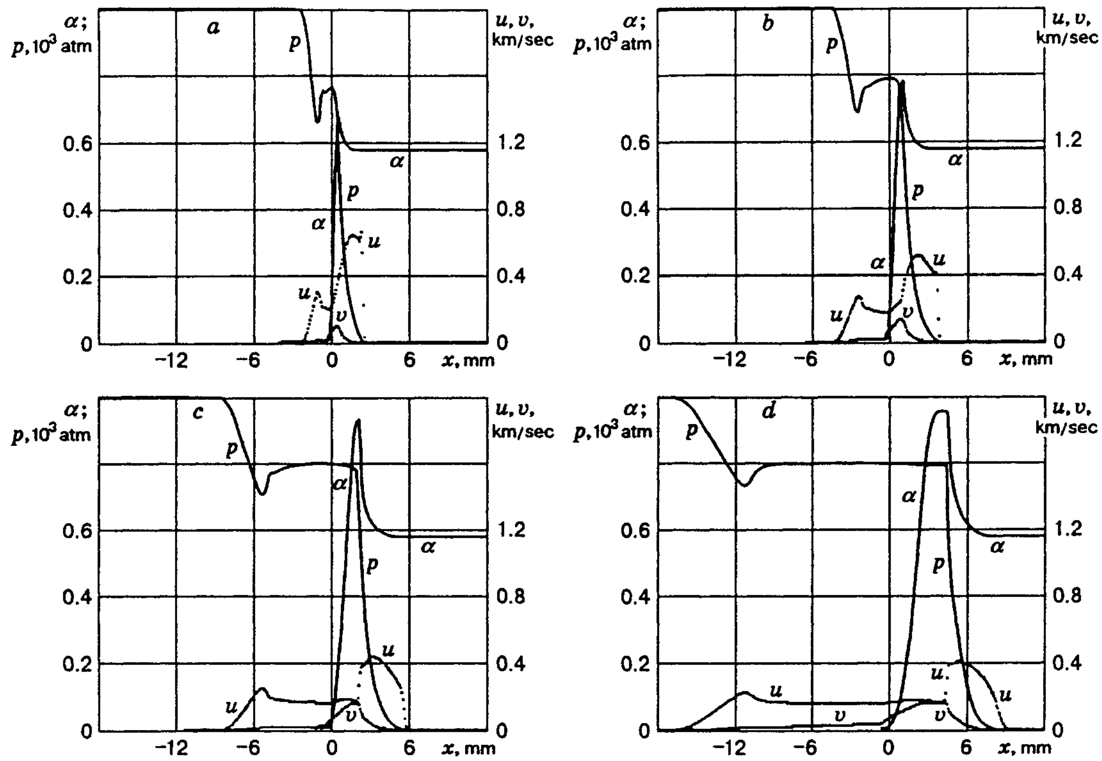


Fig. 3

holding a second-order approximation. The time step was chosen from the Courant condition and also from the stability condition for the right sides. Weak two-phase instability was eliminated by smoothing of the flow parameters, which is equivalent to a small artificial viscosity [5, 6].

Figure 3 shows the flow evolution for $p_0 = 10^3$ atm, $\rho_0 = 0.1$ g/cm$^3$, and $\gamma = 1.67$, ignoring the "solid" pressure $p_s$. The porous medium consists of particles with a size of 0.3 mm and has a density of 2 g/cm$^3$ and an initial porosity $\varphi_0 = 0.42$. Graphs of the pressure $p$, the volume fraction of the solid phase $\alpha$, the gas velocity $u$, and the particle velocity $v$ for times of 1.25, 2.5, 5, and 10 $\mu$sec (Fig. 3a–3d) are given. To simplify the calculation, we assume that in the gas region ($x < 0$) there is a low particle concentration ($\alpha \approx 0.005$) which does not influence motion of the gas. Therefore, the particle velocity was also defined for $x < 0$.

Figure 3 shows penetration of the gas into the pores, gradual particle acceleration, and progress of compaction near the boundary of the porous medium. A rarefaction wave moves to the left in the gas, and, as a result, the pressure on the boundary of the porous medium is lower than the initial pressure ($\approx 0.8p_0$). This small correction does not influence estimates. The minimum near the wave edge results from the more intense filtration in the initial period.

The calculation also shows some features of the processes that are not detected by the approximate model: (1) although the solid pressure is ignored, the final compaction is limited and (2) the peak density of the solid phase is attained at a certain depth rather than at the interface. Both of these circumstances are explained by gas penetration into the pores. As a result, having a high density, the medium begins to resist compression due to the elasticity of the gas entrapped in the pores (as is seen in Figs. 3b and 3c, the medium in the compressed region can be considered one-velocity). This stage is characterized by perturbation velocity $\approx \sqrt{\gamma p/\rho\varphi}$ ($\varphi$ is the current porosity, and $\rho$ is the total density) with which the compression wave peak moves.

Upon reaching maximum compression, the compaction zone becomes impermeable. Its thickness grows under gas action (Fig. 3d). Almost the entire pressure gradient falls on the region of particle acceleration ahead of the wave. For the wave velocity $D$ at this stage, estimation using the conservation laws gives a value of $D = \sqrt{p/\rho_s \alpha_0 \varphi_0}$, which agrees well with the calculated value.

The stage of pure filtration is not long; filtration and compaction are not separated in time. Nevertheless, estimate (8) agrees well with numerical calculations: time $t_c = 4.8$ $\mu$sec from (8) and $\approx 5$ $\mu$sec from the calculations. Thus, the simplified two-step model describes with a reasonable approximation the leading edges of the waves $v(x,t)$ and $\alpha(x,t)$ and the motion of the gas before collapse.

Allowance for the solid pressure does not introduce a marked difference if $p_s \leqslant p_0$ under maximum compression. On the other hand, allowing for the heat exchange of the phases is necessary. When the heat exchange $q$ is ignored, the gas temperature increases to unreal values because of friction. This, in turn, changes the flow pattern so that filtration proceeds more rapidly, and compaction is inhibited. Interestingly, the simple model (4) is closer to reality than the fuller system (1), whose single disadvantage is the absence of heat exchange (although this simplification would seem to be natural for the "adiabatic" equation of state).

**Comparison of Calculations with the Experiment.** It is known that gas leak in explosions in a porous medium has a significant effect on the work performed by the explosion [2–4]. According to estimates, the portion of the gas that escapes from the explosion cavity in the initial stage amounts to tens of percents.

We consider the experiments of [11], in which a PETN charge with radius $a_c$ was exploded inside spherical cavities in sand with different initial radius $a$ ($1 \leqslant a/a_c \leqslant 6.13$). In this case, the initial gas density $\rho_0$ is varied. From (9) we can estimate the mass loss:

$$\frac{\Delta m}{m} \approx \frac{3d}{a} 2\varphi_0^2 \left(\frac{\rho_s}{\rho_0}\right)^{2/3} \approx \frac{3d}{a_c} 2\varphi_0^2 \left(\frac{\rho_s}{\rho_c}\right)^{2/3} \frac{a}{a_c}.$$

Here $\rho_c$ is the HE density. For the data in [11] ($\rho_c = 1.4$ g/cm$^3$, $\varphi_0\rho_s = 1.58$ g/cm$^3$, $d = 0.25$ mm, and $a_c = 5.13$ mm), we have $\Delta m/m \approx 0.1a/a_c$. The results of [11] for $1 < a/a_c < 3$ can be represented as $\Delta m/m \approx 0.2 + 0.2a/a_c$. Note that, for a small initial radius of the cavity ($a/a_c \approx 1$), the gas density is comparable with the particle density, and, for large radii, the medium's strength has an effect. For moderate radii, the results reflect qualitatively the experimental dependence and agree numerically in order of magnitude. It should also be taken into account that the data of [11] are obtained for the maximum cavity radius, which was compared with the calculated radius in the absence of filtration. It would be of interest to measure directly the gas leak and the time of collapse of pores.

The effects described are very important for the initiation of powdered HE upon contact with a hot gas. Unfortunately, no direct data are available for comparison, although, there is indirect evidence for a possible relationship between compaction and accelerated explosion initiation. These questions are discussed in detail in [1].

Thus, we constructed an approximate model for the compaction of a brittle powder by a gas filtering into pores that agrees qualitatively with numerical calculation. The model can be useful in estimating parameters such as the time of compaction and the amount of gas injected ahead of the compression wave of the porous bed.

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